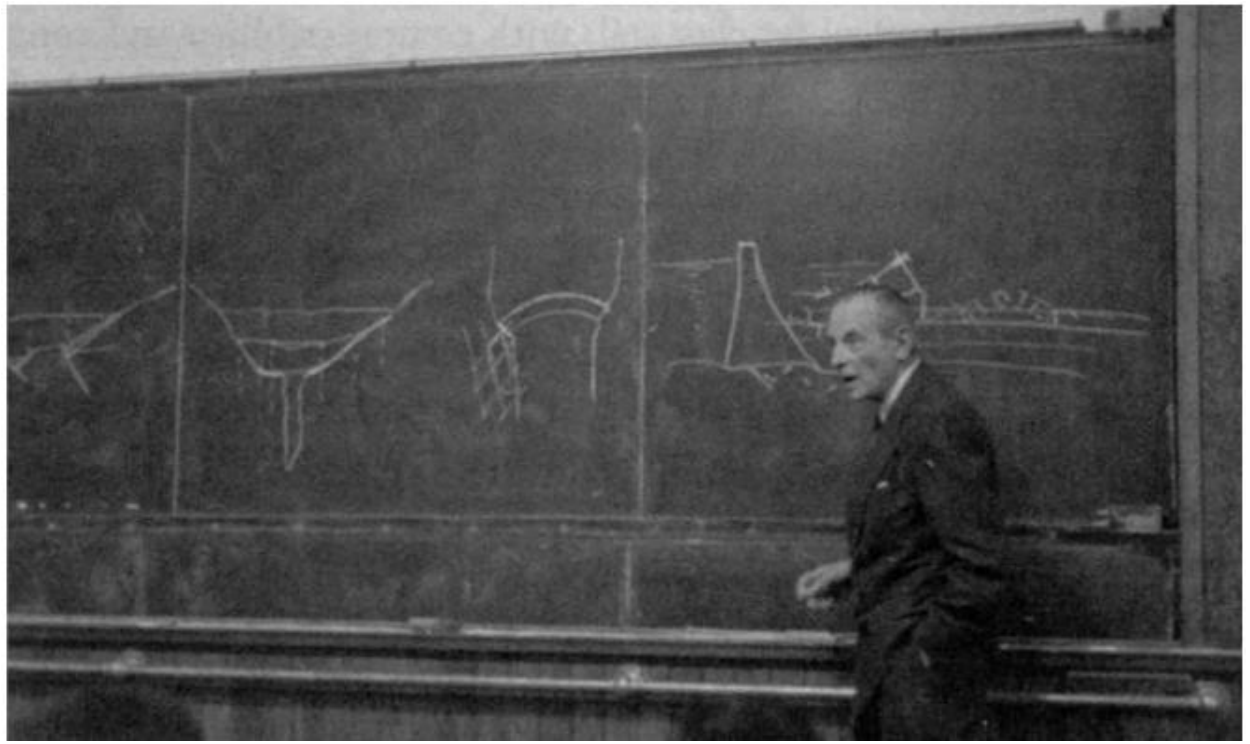
THE SAFETY OF SLOPES

D.V. Griffiths
Colorado School of Mines



Karl Terzaghi lecturing at Harvard in 1950 Photo from de Boer (2005)

KU School of Engineering
58th Annual Geotechnical Engineering Conference

November 12th, 2026

Karl Terzaghi Lecturers



1963	Peck	1984	Mitchell	2005	Fredlund
1964	Casagrande	1985	Osterberg	2006	Krizek
1965		1986	Ladd	2007	Goble
1966	Bjerrum	1987	Zeevaert	2008	Giroud
1967	Seed	1988	D'Appolonia	2009	Baker
1968	Rutledge	1989	Schmertmann	2010	Holtz
1969	Wilson	1990	Gould	2011	Stokoe
1970	Lambe	1991	Duncan	2012	Daniel
1971	Lowe	1992	Morgenstern	2013	Hendron
1972	McClelland	1993	Focht	2014	Santamarina
1973		1994	Clough	2015	Bruce
1974	Richart	1995	Olson	2016	O'Rourke
1975	Meyerhof	1996	Koerner	2017	Rowe
1976	Reese	1997	Woods	2018	Bonaparte
1977	Legget	1998	O'Neill	2019	Idriss
1978	Newmark	1999	Marcuson	2020	Cording
1979	Sowers	2000	Hoek	2021	Baecher
1980	Leonards	2001	Lacasse	2022	Kavazanjian
1981	Whitman	2002	Milligan	2023	Mayne
1982	Cooke	2003	Christian	2024	Whittle
1983	Scott	2004	Poulos	2025	Springman

THE SAFETY OF SLOPES

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I. METHODS OF ASSESSING THE SAFETY OF SLOPES

- The Factor of Safety (*FS*) and the Probability of Failure (p_f)

II. THE CLASSICAL APPROACH (*FS*)

- Charts
- Methods of slices
- Numerical methods (FE/FD)
  - Critical Pool Level (*FS* minimum)

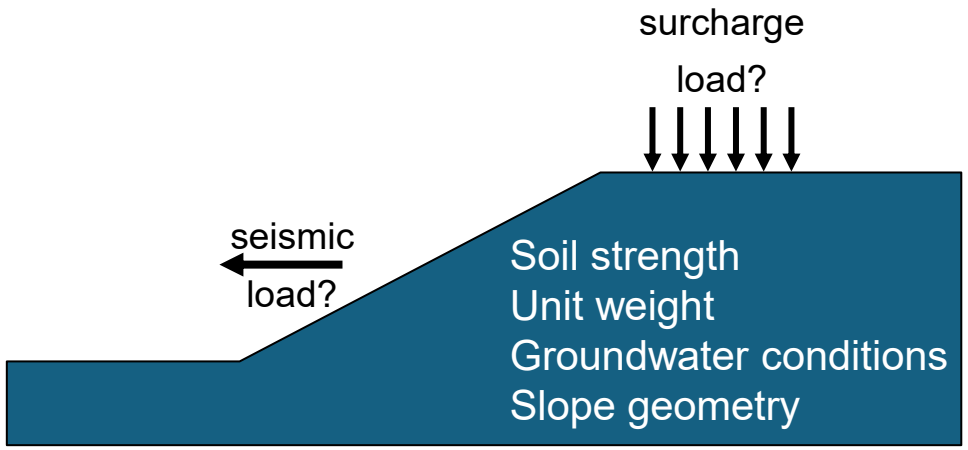
III. THE PROBABILISTIC APPROACH (p_f)

- SRV method for finding p_f
- RFEM for finding p_f (including spatial variability)
  - Worst-case spatial correlation length (p_f maximum)

IV. CONCLUDING REMARKS

I. METHODS OF ASSESSING THE SAFETY OF SLOPES

- **The Factor of Safety (FS) and the Probability of Failure (p_f)**



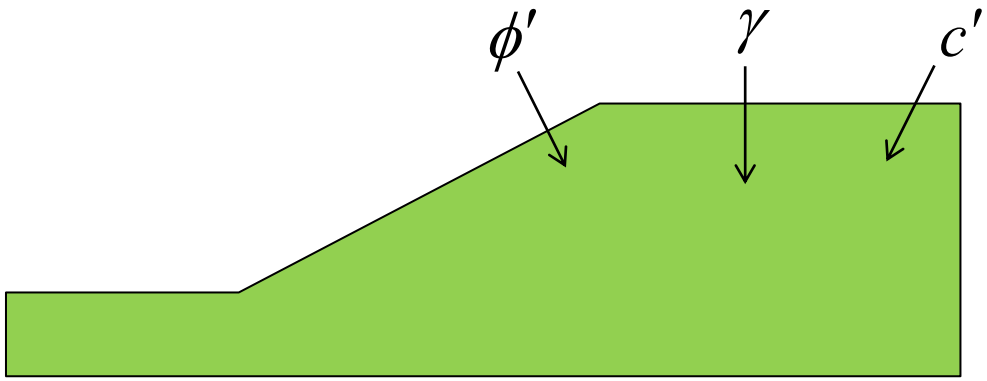
The Factor of Safety (FS) is the classical measure of the margin of safety
e.g. if *FS* =1.5 this means the strength of the soil is 50% higher than needed to maintain equilibrium with gravity and any other external loads.
The Factor of Safety is a "strength reduction factor".

The Probability of Failure (p_f) is an alternative measure of the margin of safety,
e.g. if p_f =1% and a hundred such slopes were built, one of them would be expected to fail during the design life.

Main Challenges
- Characterizing the soil strength and groundwater conditions
- Finding the critical failure mechanism

II. THE CLASSICAL APPROACH (*FS*)

Input requires "characteristic values" which are essentially cautious estimates of strength parameters based on numerous factors including the level of knowledge of the site and consequences of failure.



Perform Slope Stability Analysis

- Charts
- Limit equilibrium (Methods of slices)
- Numerical Methods, e.g. Finite Elements/Differences

↓

FS

↓

Design criterion: $FS \geq FS_{target}$?

Methods for finding FS

SLOPE CHARTS

e.g. Taylor's Chart (1937)

For undrained slopes

Taylor's chart delivers a *Stability Number*

based only on slope geometry

$$N_s = f(\beta, D)$$

assuming **circular failure mechanisms**

from which the Factor of Safety is given by

$$FS = \frac{s_u}{\gamma H} N_s$$

Note:

$$FS \propto s_u$$

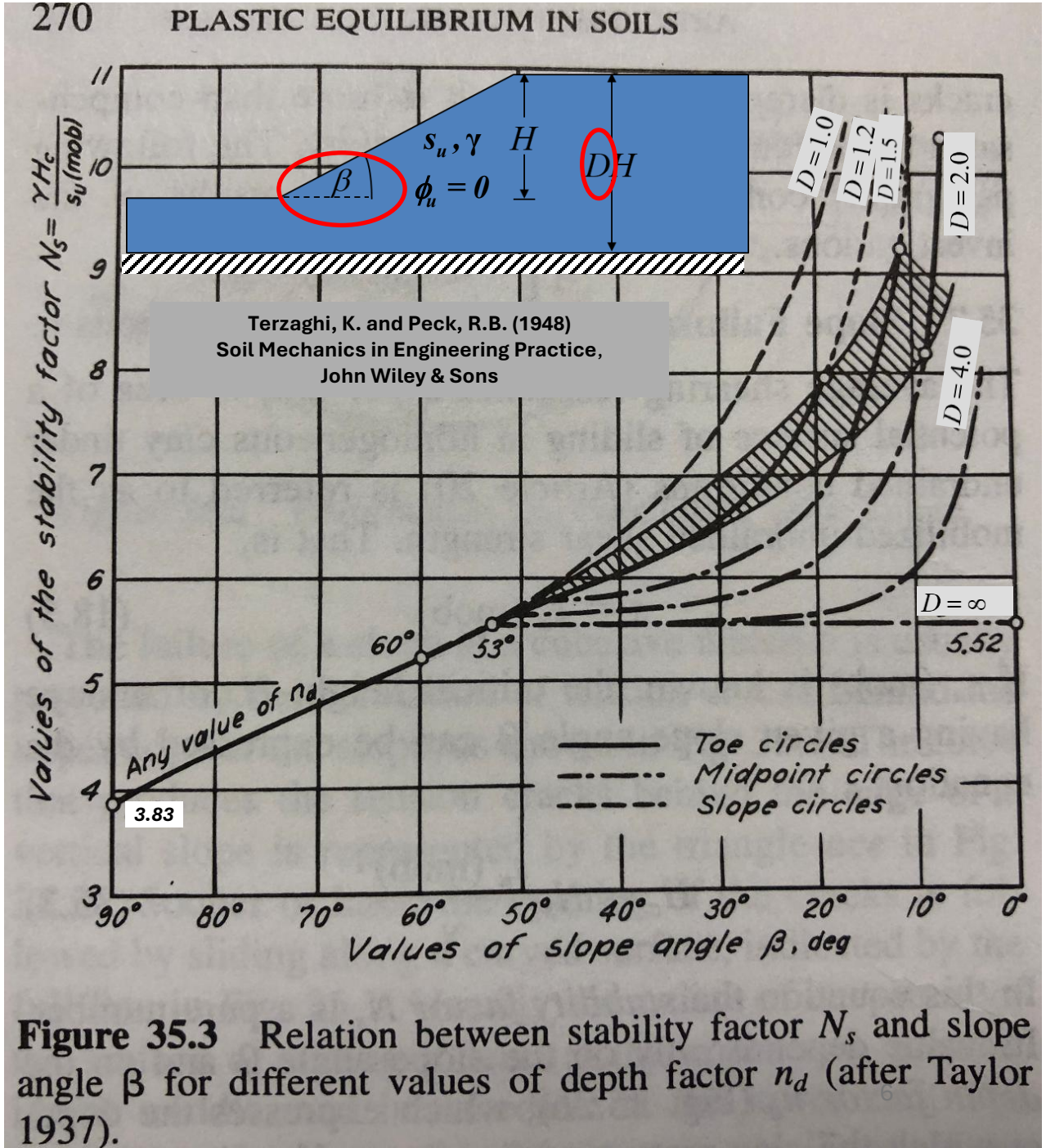


Figure 35.3 Relation between stability factor N_s and slope angle β for different values of depth factor n_d (after Taylor 1937).

It is not clear how Taylor created this chart in 1937

For every combination of β and D, Taylor must have manually searched for the circular failure mechanism that gave the lowest Factor of Safety!

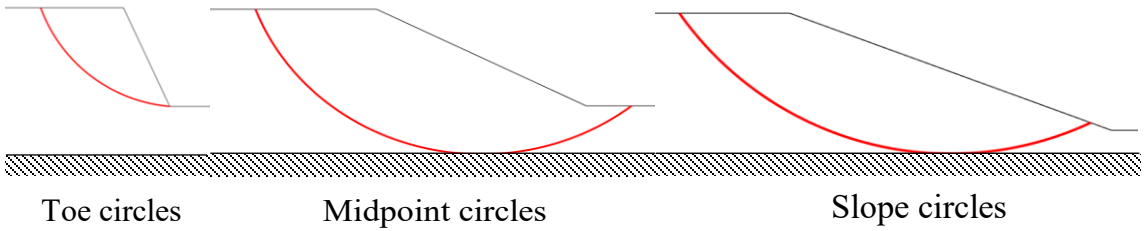
In his seminal textbook

***FUNDAMENTALS OF SOIL MECHANICS*, p.455,**

Taylor concedes that :

"A considerable amount of labor is required to obtain such charts, but once obtained, solutions by their use are simple indeed."

The chart highlights three types of circular failure mechanisms for flatter slopes, but for steeper slopes $(\beta \geq 53°)$ the failure mechanism always passes through the toe.



Taylor would be impressed with routine software available nowadays, that can search for the critical cicular mechanism and corresponding *FS* almost instantaneously.

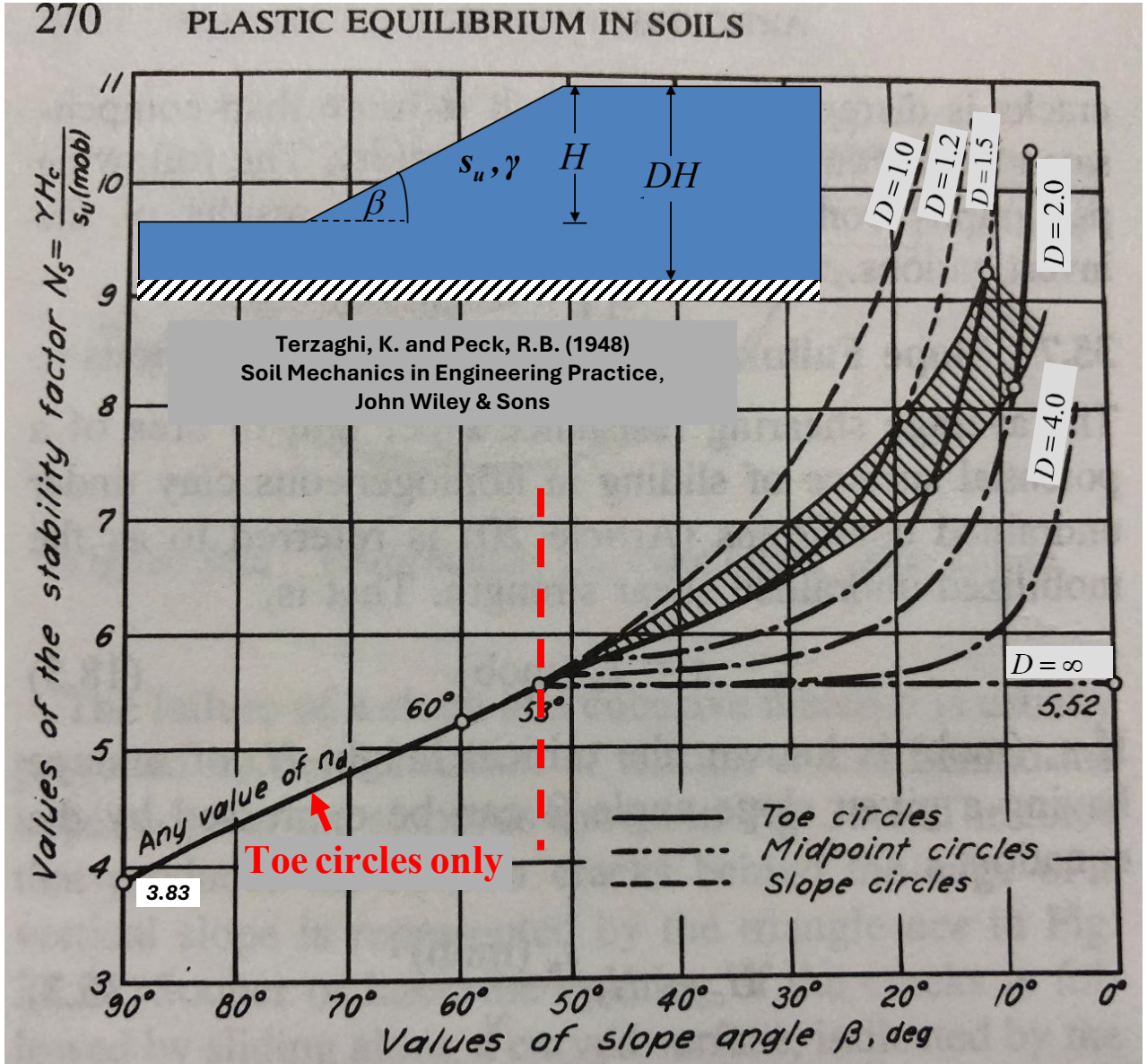
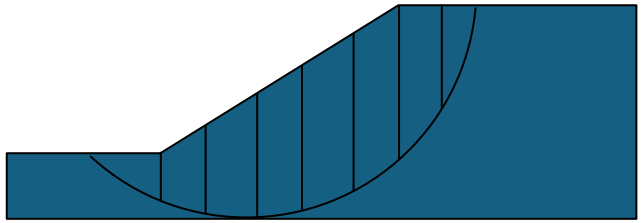


Figure 35.3 Relation between stability factor N_s and slope angle β for different values of depth factor n_d (after Taylor 1937).

LIMIT EQUILIBRIUM METHODS

Refer to a multitude of "Methods of Slices" dating back to Fellenius in the 1930s.



Widely used in geotechnical practice and incorporated into numerous proprietary codes.

Which method should be used?

Typical menu...

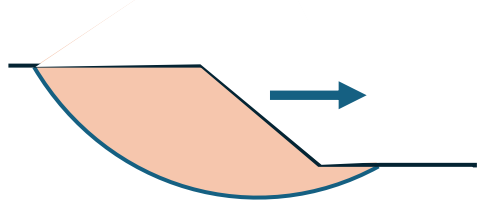
Morgenstern-Price
Spencer
Corps of Engineers #1
Corps of Engineers #2
Lowe-Karafiath
Janbu Generalized
Sarma
Bishop
Janbu
Ordinary Method of Slices
.
.

Too much choice..?

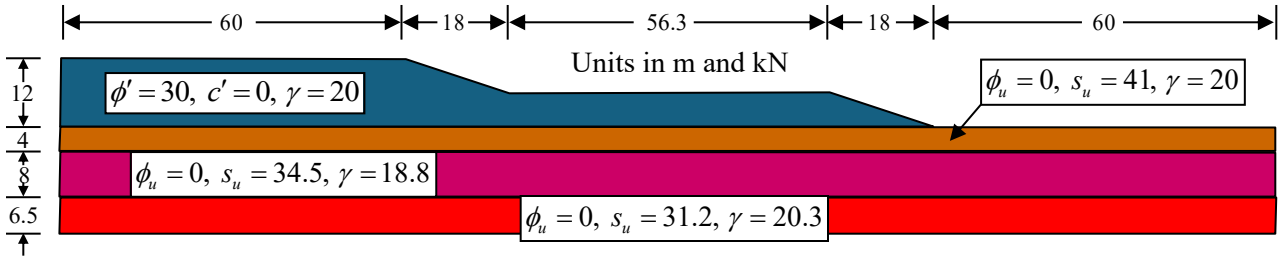
Failure Direction?

Left to right ☒

Right to left ☐

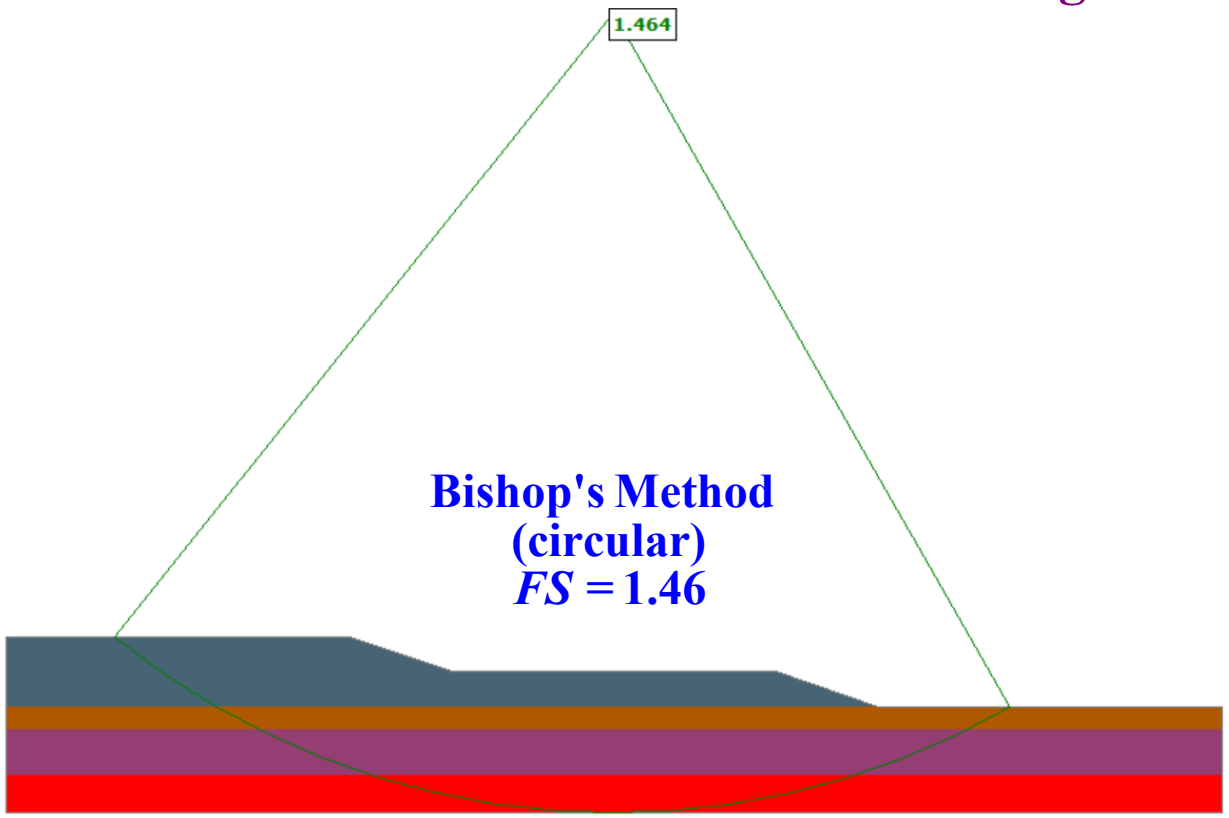


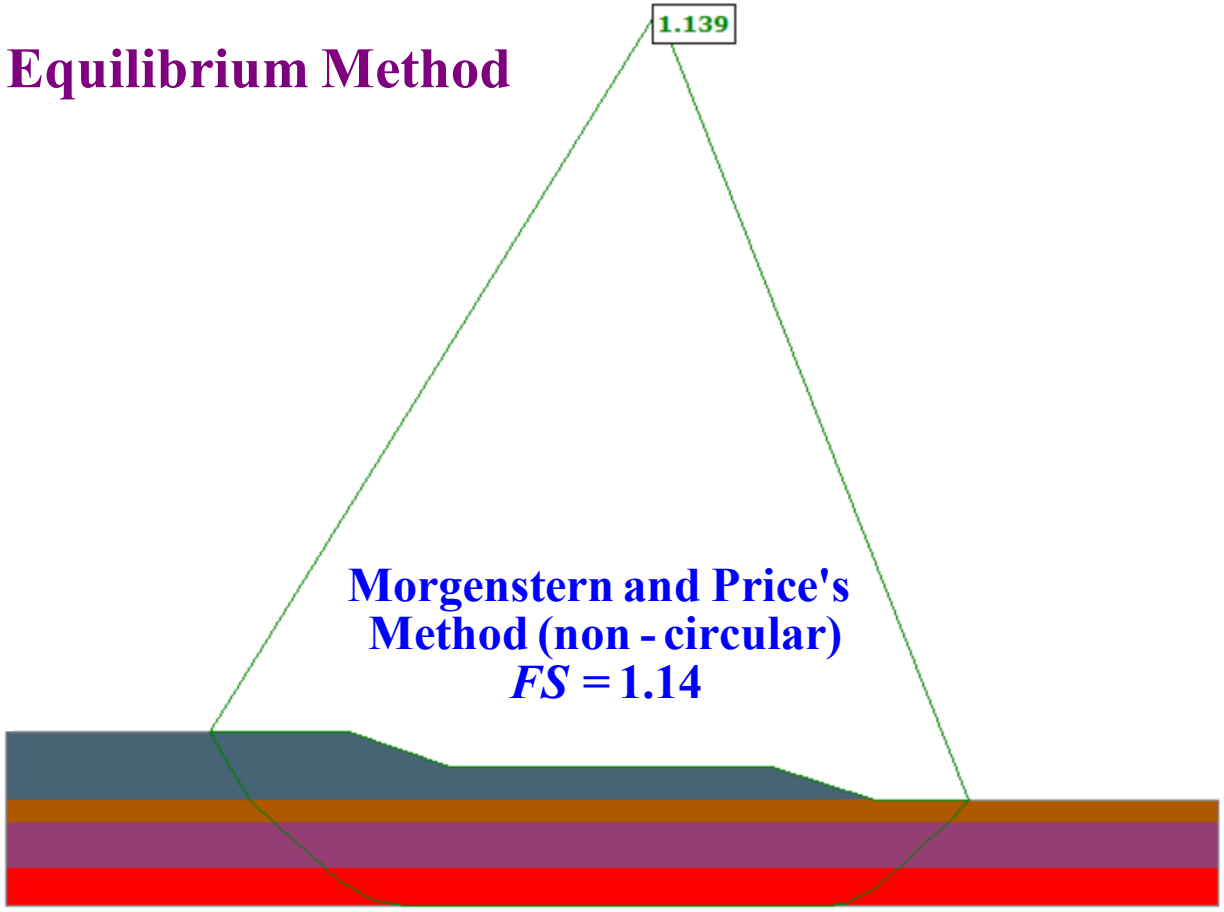
The **James Bay Dike** is a well-known test problem



Units in m and kN

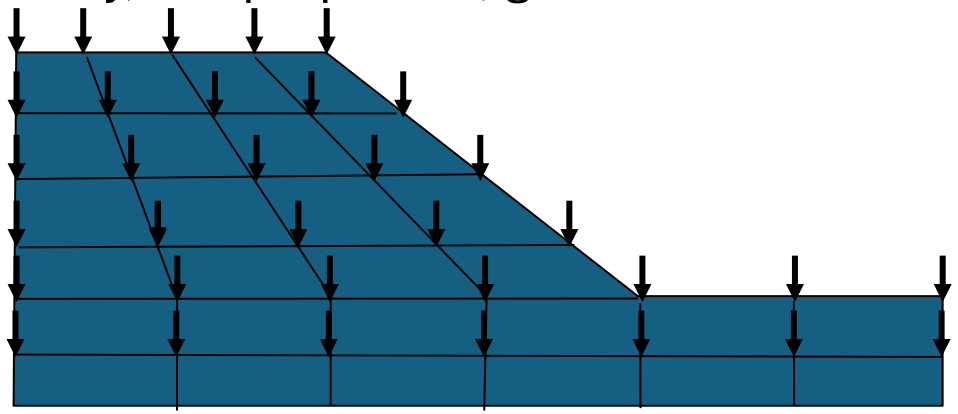
Find *FS* using a Limit Equilibrium Method



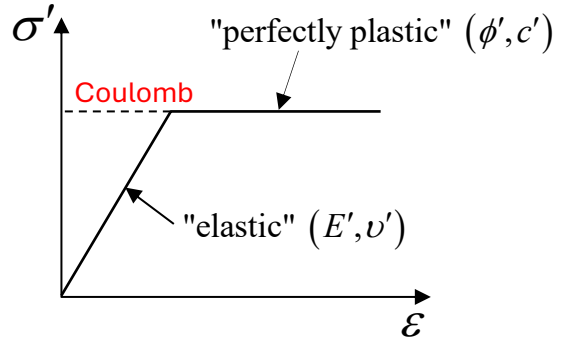


NUMERICAL METHODS: FINITE ELEMENTS WITH STRENGTH REDUCTION

- Set up a finite element mesh representing the slope geometry, soil properties, groundwater conditions etc.

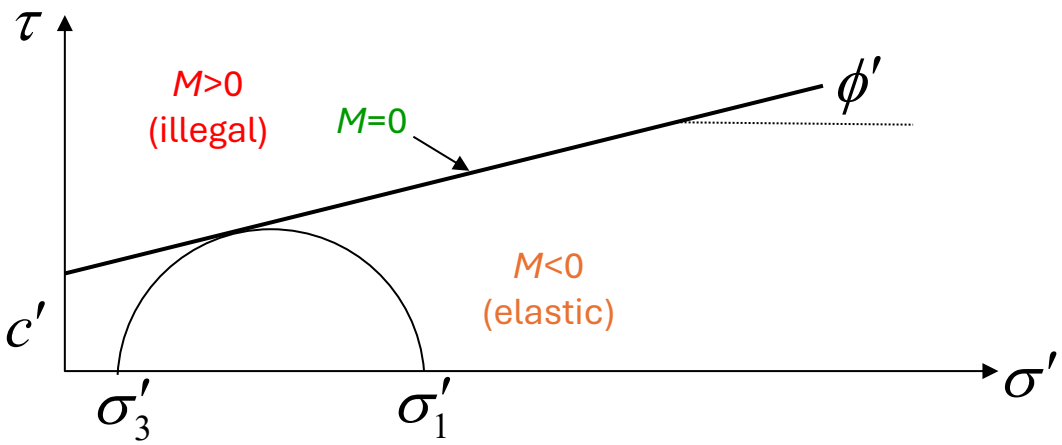


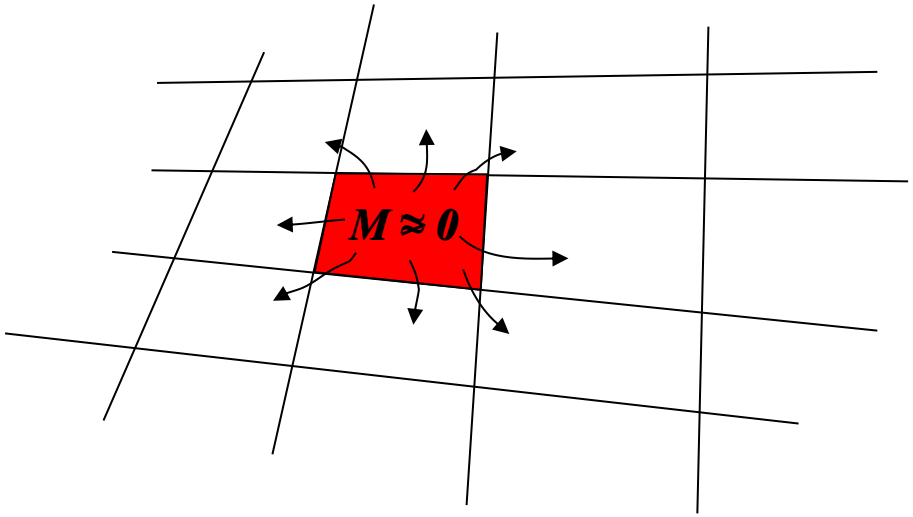
soil is given ane **elastic-perfectly plastic** stress-strain model



- Apply gravity loads to the mesh.

- Compute elastic stresses and check for elements violating Coulomb





- **Find elements with "illegal" elastic stresses violating Coulomb ($M > 0$)**
- **Redistribute excess stress to neighboring elements**
- **Method is iterative**
- **At convergence, all element should have $M \leq 0$ within tolerances.**
- **Failure to converge indicates slope failure accompanied by a sudden increase in nodal displacements**

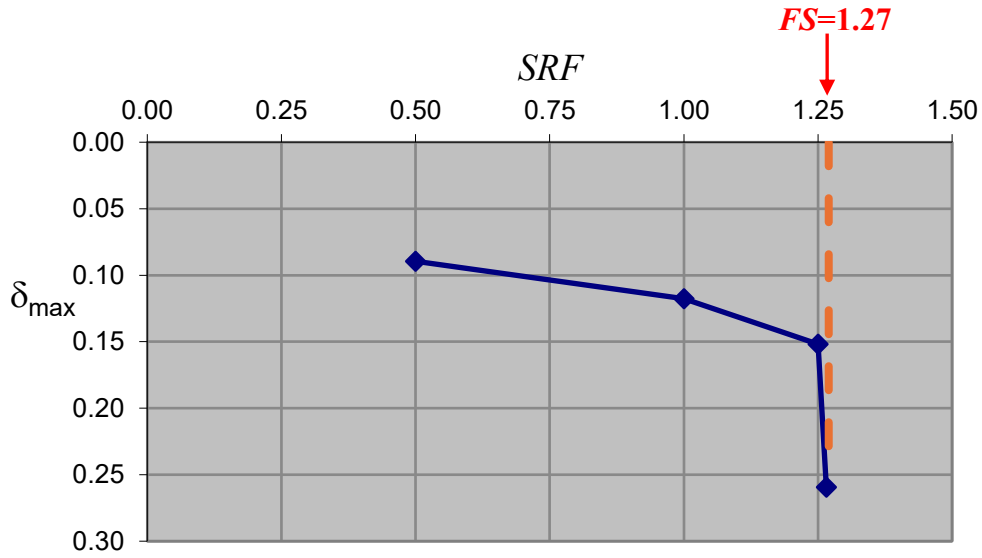
Strength reduction to failure

Originally assigned properties (c' and ϕ')

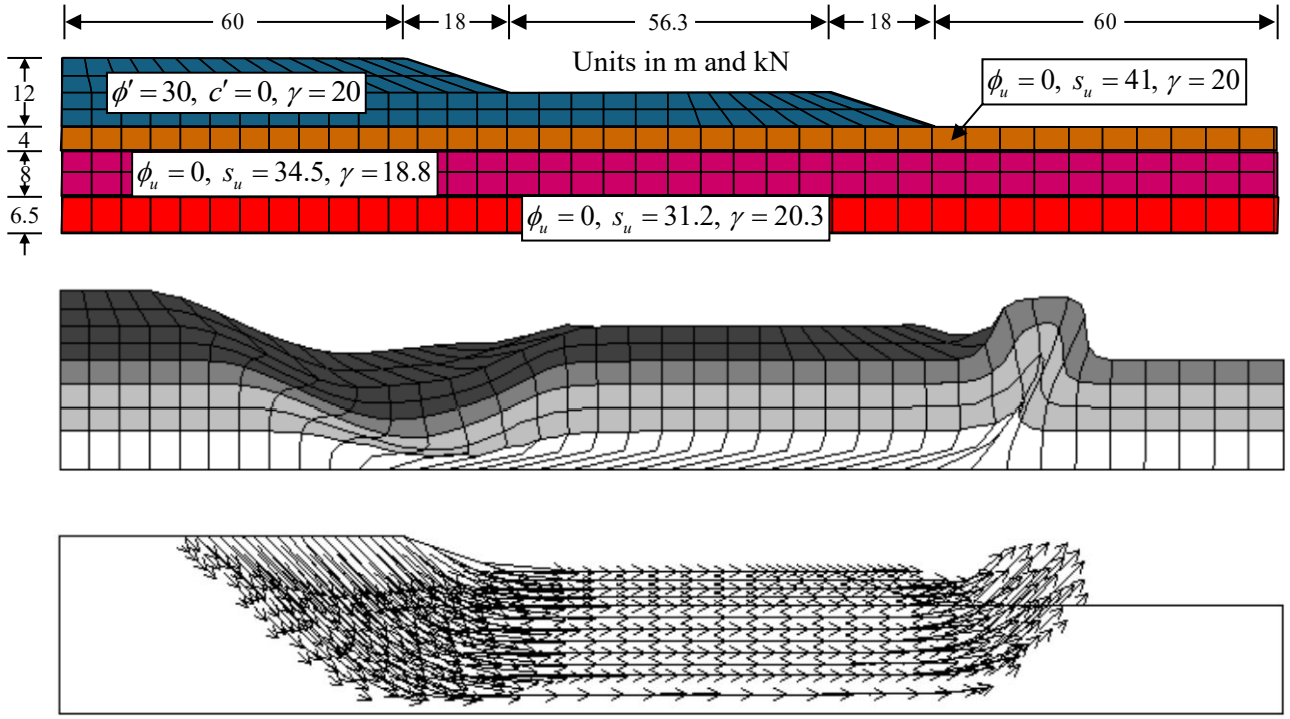
Gradually weaken the soil using a Strength Reduction Factor (SRF)

$$c'_f = \frac{c'}{SRF} \qquad \phi'_f = \arctan\left(\frac{\tan\phi'}{SRF}\right)$$

At failure $FS \approx SRF$



James Bay Dike by Finite Elements with Strength Reduction



Deformed mesh at failure

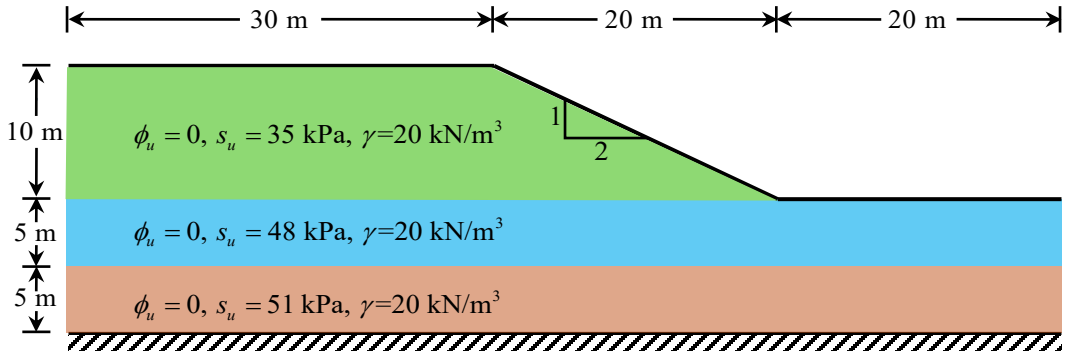
Nodal displacement vectors at failure

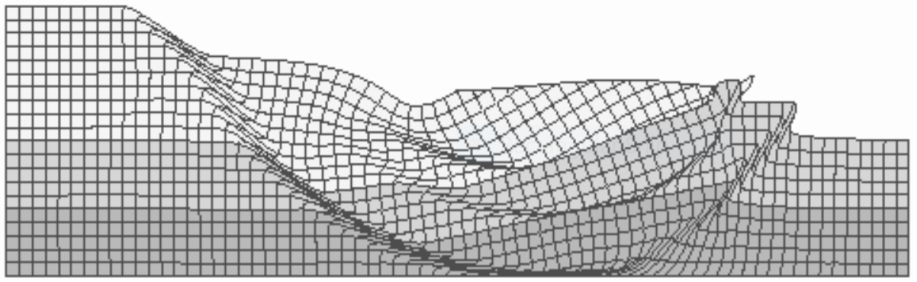
***FS*=1.27 by finite elements with strength reduction**

Recall Bishop (*FS*=1.46) and Morgenstern & Price (*FS*=1.14)

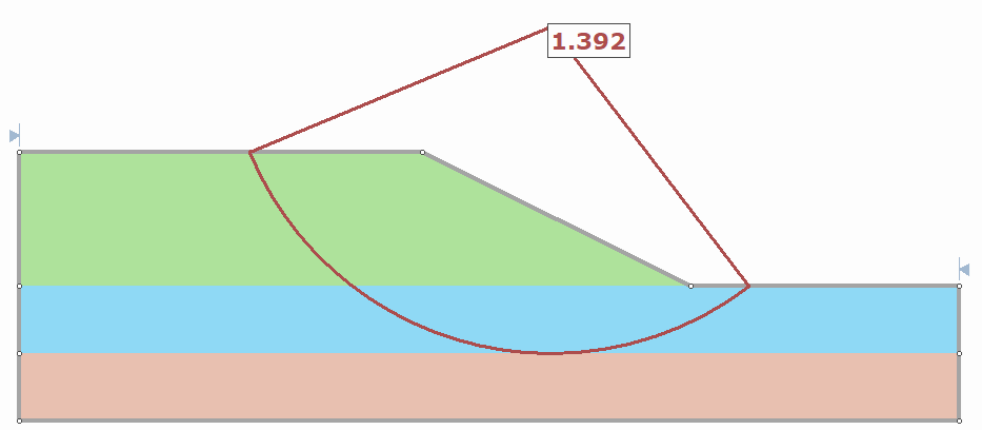
https://inside.mines.edu/~vgriffit/slope64

Finite elements with strength reduction automatically finds multiple mechanisms if they exist.





FE with strength reducution finds three mechanisms with $FS \approx 1.38$



Limit equilibrium methods find one mechanism with the lowest Factor of Safety

FINITE ELEMENT LIMIT ANALYSIS

Derived from the theorems of plasticity which state that:

1) A statically admissible stress field that everywhere satisfies equilibrium and does not violate the failure criterion will always give a LOWER BOUND,

i.e. the predicted failure load is ≤ the exact failure load

2) A kinematically admissible failure mechanism that does not violate the failure criterion will always give an UPPER BOUND,

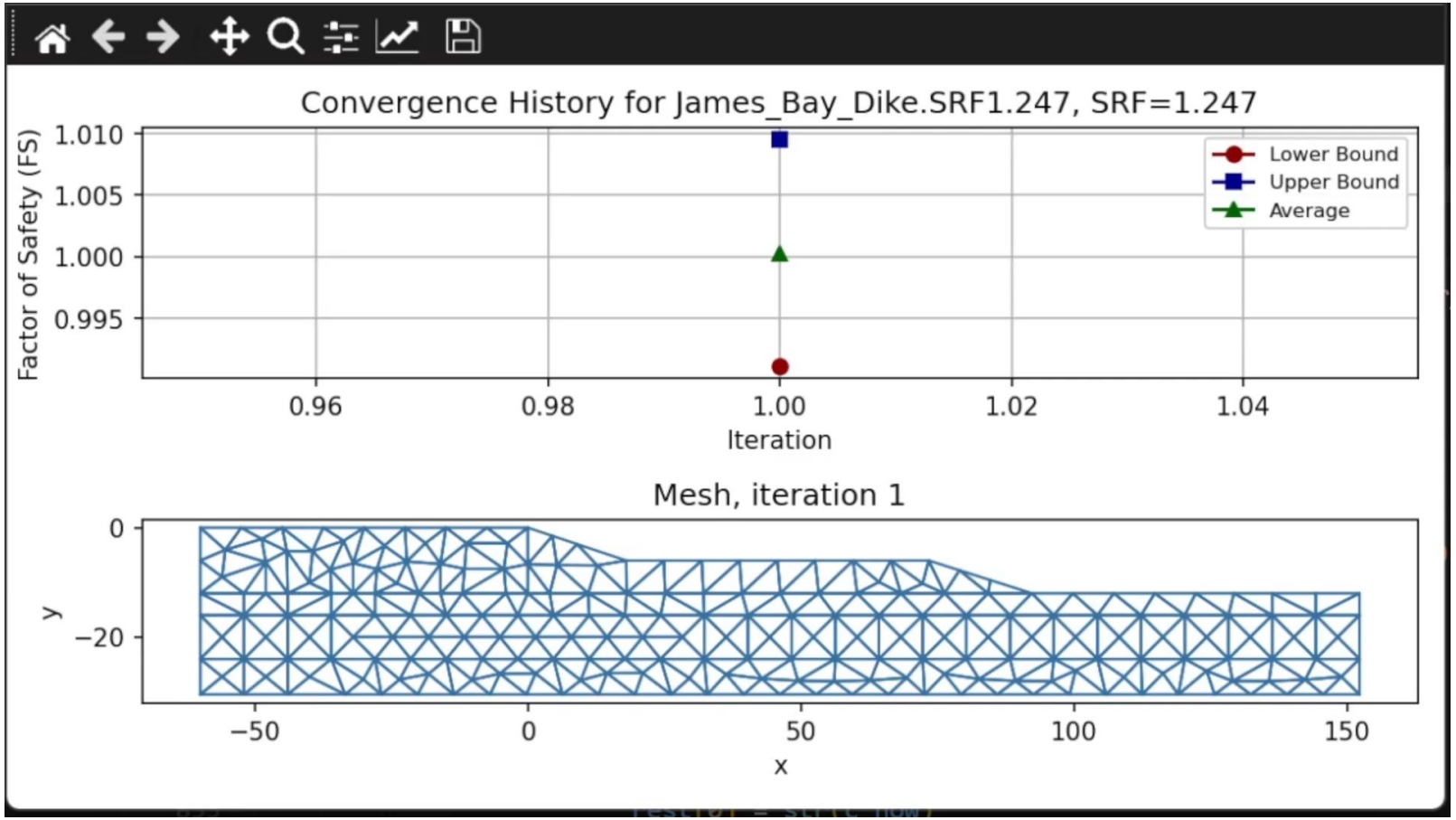
i.e. the predicted failure load is ≥ the exact failure load

$$\underset{\text{Failure Load}}{\text{Lower Bound}} \leq \underset{\text{Failure Load}}{\text{Exact}} \leq \underset{\text{Failure Load}}{\text{Upper Bound}}$$

The goal of a limit analysis is to obtain a lower and upper bound that are close together

James Bay Dike by Finite Element Limit Analysis

Oxlim2d developed by Martin (2011)
Uses "adaptive Mesh Refinement"

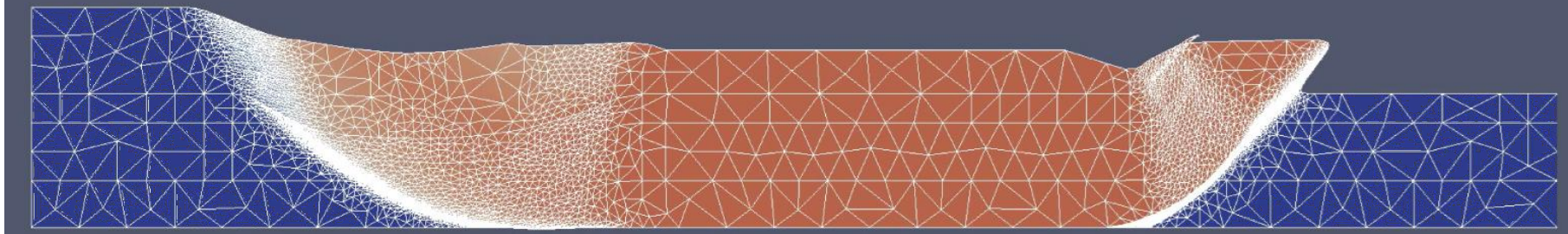


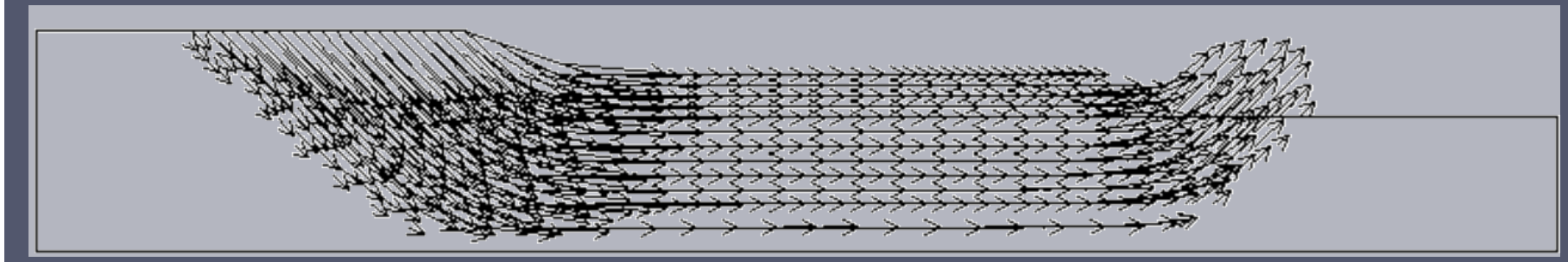
***FS*=1.25 by limit analysis**

Advantages of both Finite Element approaches:

- **The failure mechanism "seeks out" the path of least resistance and reports the corresponding Factor of Safety.**
- **Easier for the user. No choices to be made about the "method" to be used, the shape or location of the critical failure surface, direction of failure, etc.**

Finite Element Limit Analysis : $FS = 1.25$





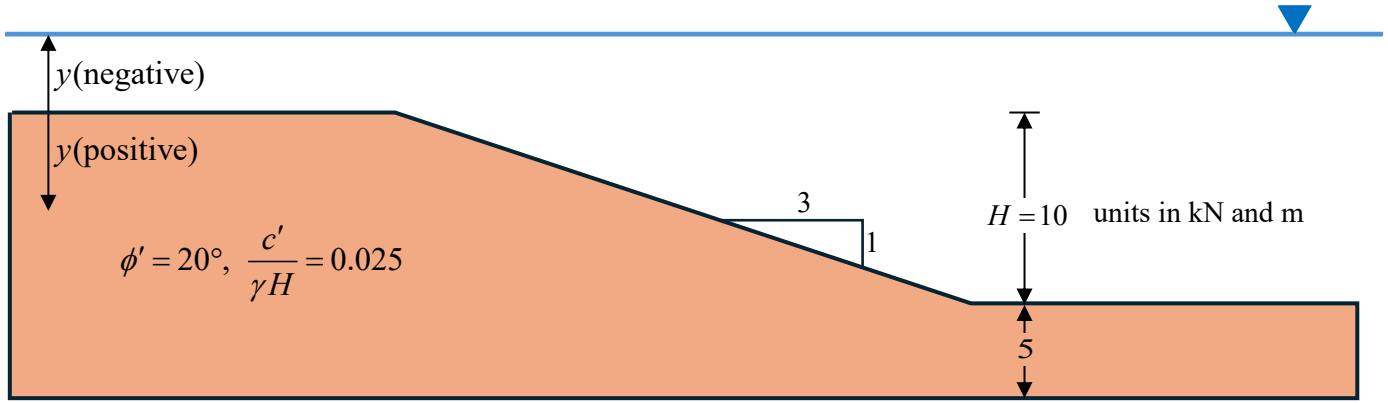
Finite Elements with Strength Reduction : $FS = 1.27$

- **Critical pool level (*FS* minimum)**

"On a planet without any water there would be no need for soil mechanics"
Karl Terzaghi, Proc. Inst. C.E., 12: 106-141 (1939)

Is water always bad?

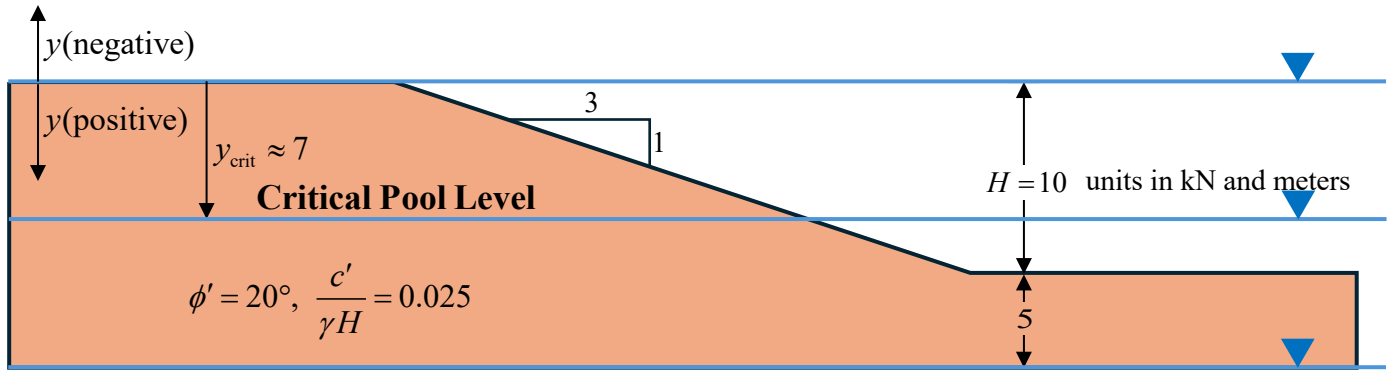
Consider a ϕ', c' slope with a horizontal water table

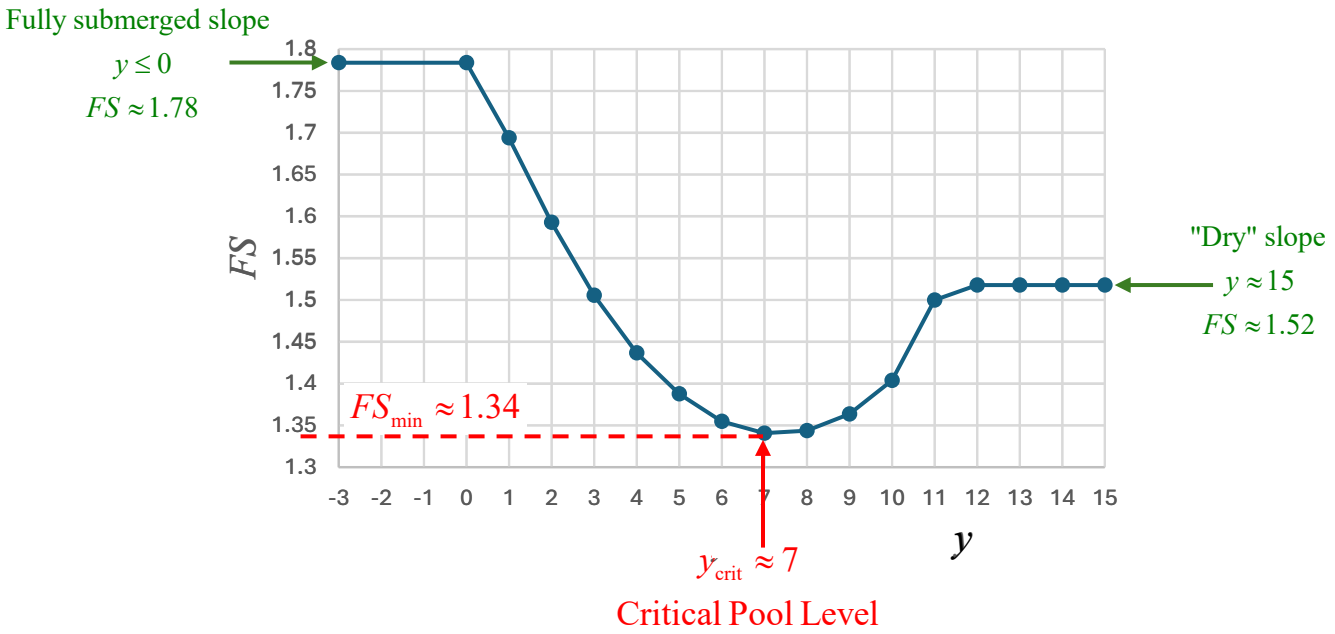


Let the water table initially be above the level of the slope (fully submerged) and then move slowly down to the bottom of the slope.

How does the *FS* change?

How does *FS* change during (slow) drawdown?





Terzaghi's principle of effective stress

$$\sigma' = \sigma - u$$

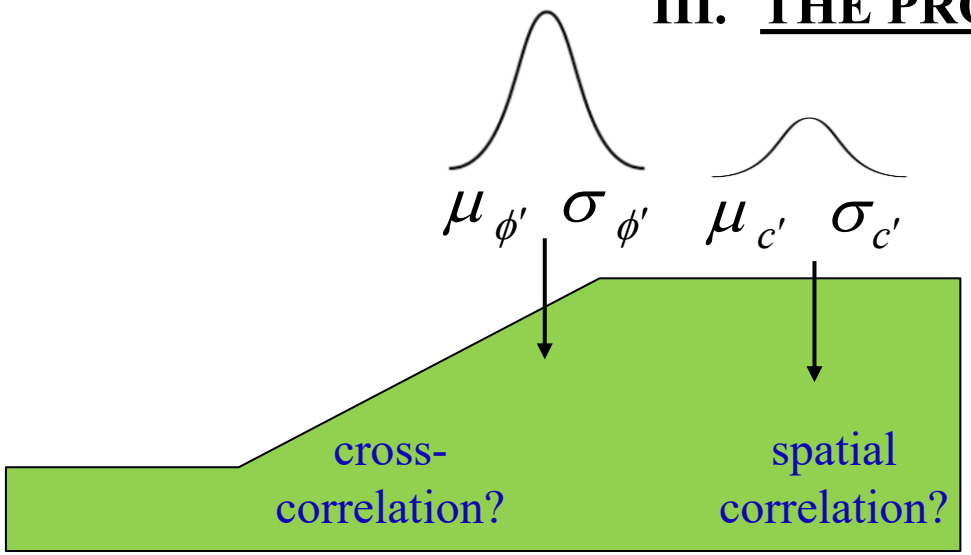
As drawdown proceeds in the range $0 < y < 15$, u is obviously falling

but in the range $0 < y < 7$

σ is falling faster than u,

so σ' and the frictional soil strength is reduced.

III. THE PROBABILISTIC APPROACH (p_f)



At least **two** parameters are needed for a p_f analysis, i.e. means and standard deviations

Note that only **one** parameter was needed for an *FS* analysis, i.e. characteristic values

Correlation properties may be needed in a more advanced analysis

Perform Probabilistic Slope Stability Analysis

- First Order Methods (FOSM, FORM)
- SRV/Monte-Carlo
- Random Finite Element Method (RFEM)

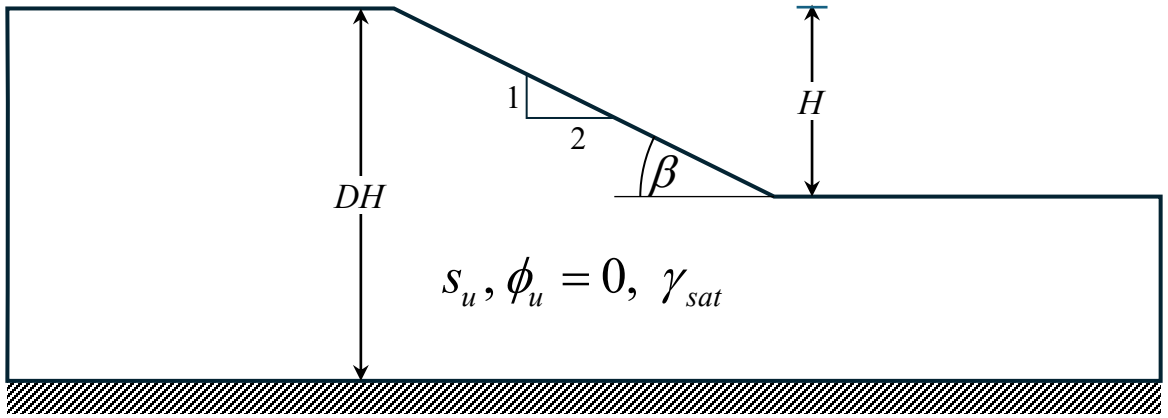
$\longrightarrow p_f \longrightarrow$ **Design criterion:** $p_f \leq p_{f\,(\text{target})}$?

$p_{f\,(\text{target})}$ is a painful concept, because it forces us to admit that there is an **acceptable probability of failure**!

The choice of $p_{f\,(\text{target})}$ is site/project specific, depending on **budget** and **consequences of failure**

Single Random Variable (SRV) method for finding p_f

First consider a uniform undrained slope



Let β=26.6°, $D = 2$, $s_u = 50$, $H = 10$ and $\gamma_{sat} = 20$ (units in m and kN)

Taylor's charts for this case gives N_s=5.89

$$\text{Hence } FS = \frac{N_s}{\gamma_{sat} H} s_u = 1.47$$

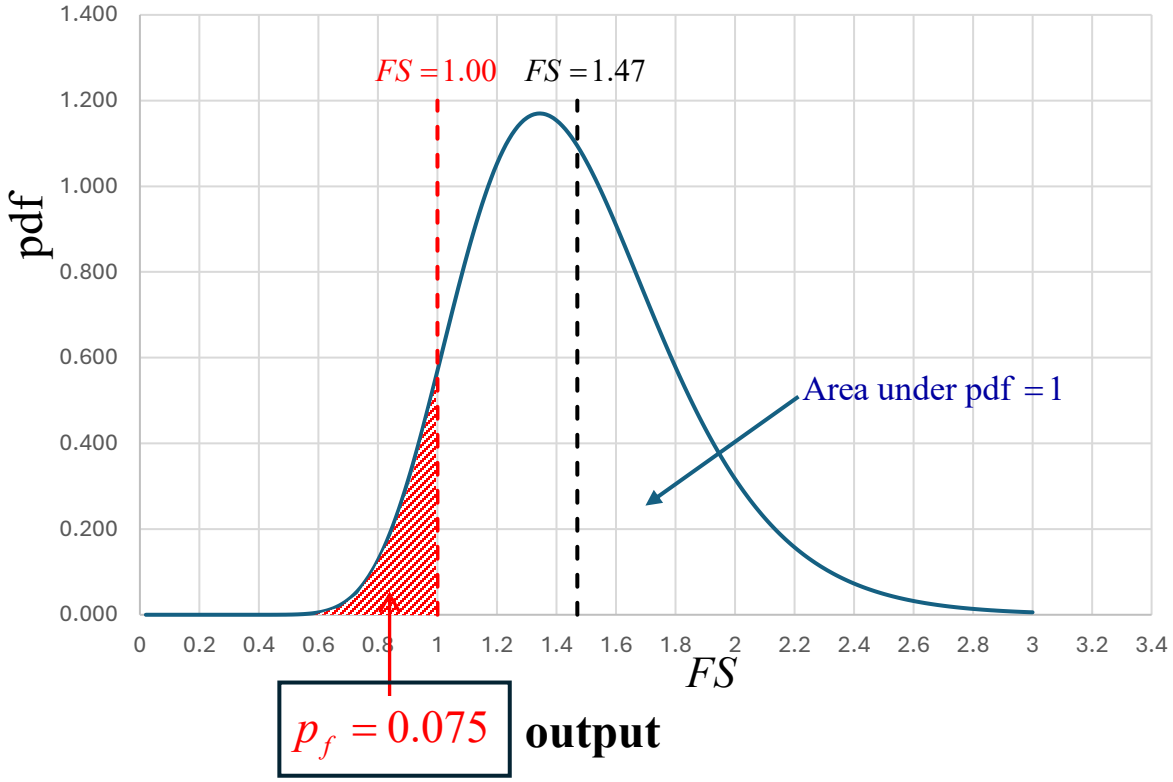
Note that $FS \propto s_u$

Now consider the same slope, but with a random shear strength s_u

Let s_u be *lognormally* distributed with $\mu_{s_u}=50$ and $\sigma_{s_u}=12.5$ **input**

i.e. a Coefficient of Variation of $v_{s_u}=\sigma_{s_u}/\mu_{s_u}=0.25$

FS $\propto s_u$ **hence** $v_{FS}=v_{s_u}=0.25$, **and** $\mu_{FS}=1.47, \sigma_{FS}=0.368$



$p_f=0.075$ **output**

The probability of slope failure p_f is given by the area under the pdf corresponding to *FS* < 1

For this case, there is a closed form equation for p_f as follows:

$$p_f=\Phi\left[\frac{\ln\left(1+v_{FS}^2\right)-2\ln\left(\mu_{FS}\right)}{2\sqrt{\ln\left(1+v_{FS}^2\right)}}\right]$$

after Duncan (2000) *JGGE*, Vol. 126, No. 4

where $\Phi(\ )$ is the cumulative normal distribution function

In the SRV approach,

the slope always has a uniform strength

which can take any random value from the pdf of s_u

In this SRV analysis, all failure mechanisms will be mid - point circles according to Taylor's chart

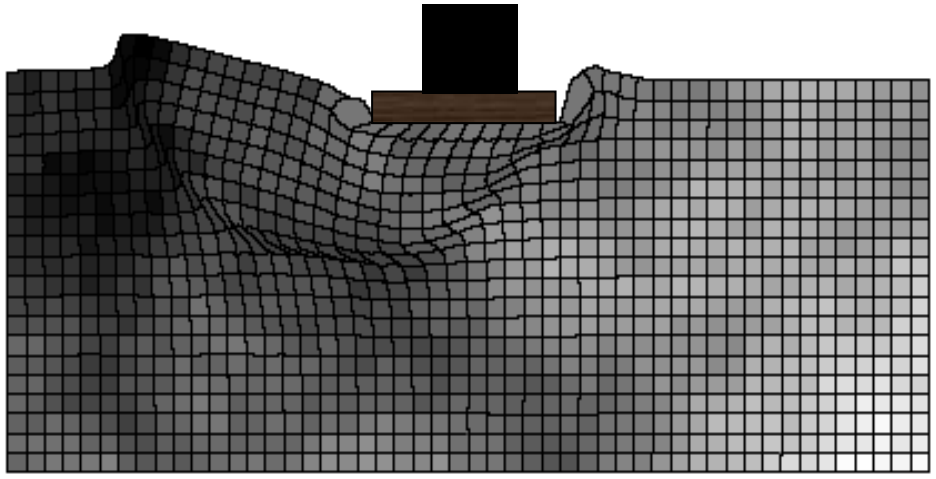
The previous example took account of the mean $\left(\mu_{s_u}\right)$ and standard deviation $\left(\sigma_{s_u}\right)$ of the shear strength **but ignored spatial variability**.

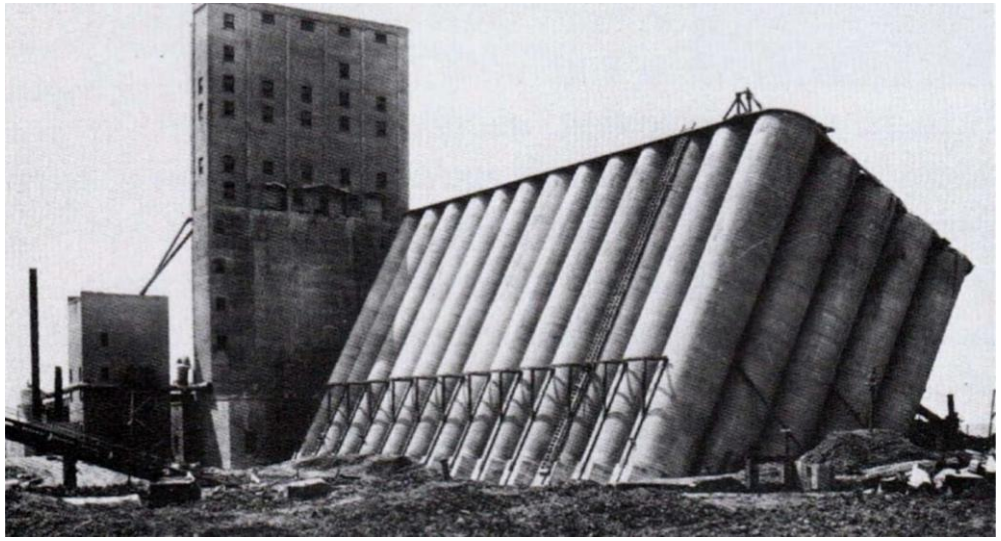
...but what is meant by **spatial variability?**

"….in earthwork engineering the designer has to deal with bodies of earth with complex structure and the properties of the material may vary from point to point."
Karl Terzaghi: Foreword to the inaugural issue of *Géotechnique*, vol.1, no.1 (1948)

It is only relatively recently however, that methodologies such as the Random Finite Element Method (RFEM) have been developed to model the variability highlighted by Terzaghi.

Bearing Capacity by RFEM





Bearing failure of the Transcona Grain Elevator, Manitoba, Canada (1913)

Bearing capacity analysis on spatially variable soil does not usually give the symmetrical mechanism assumed in Terzaghi's bearing capacity theory

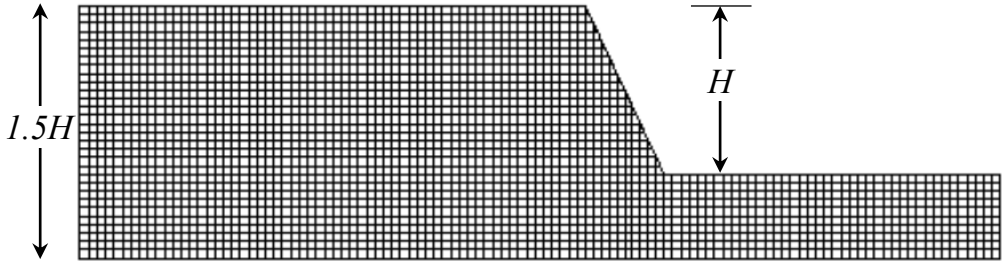
Random Finite Element Method (RFEM) for finding p_f (including spatial variability)

We need an additional property θ (units of length) called the

SPATIAL CORRELATION LENGTH

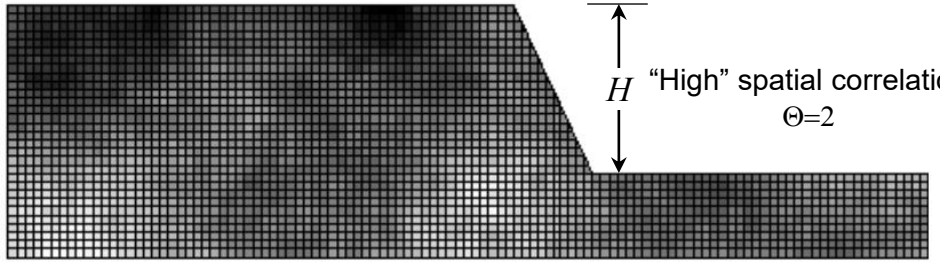
This parameter recognizes that soil samples "close together" in the field are more likely to have similar properties that if they are "far apart"

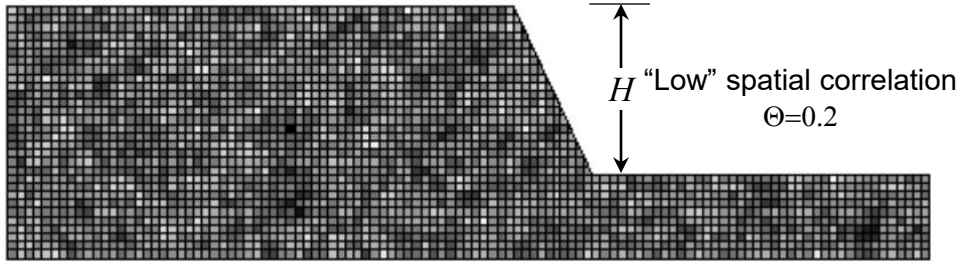
Start with a relatively fine finite element mesh



$$\Theta = \frac{\theta}{H}$$

Then map a Random Field of soil properties defined by μ, σ and Θ onto the mesh





Both slopes have the same mean and standard deviation of shear strength but quite different spatial correlation lengths.

The **Random Finite Element Method** involves repetitive **Monte-Carlo Simulations**

Generate random fields N times, each with the same statistical properties $\boxed{(\mu, \sigma, \Theta)}$ **input**

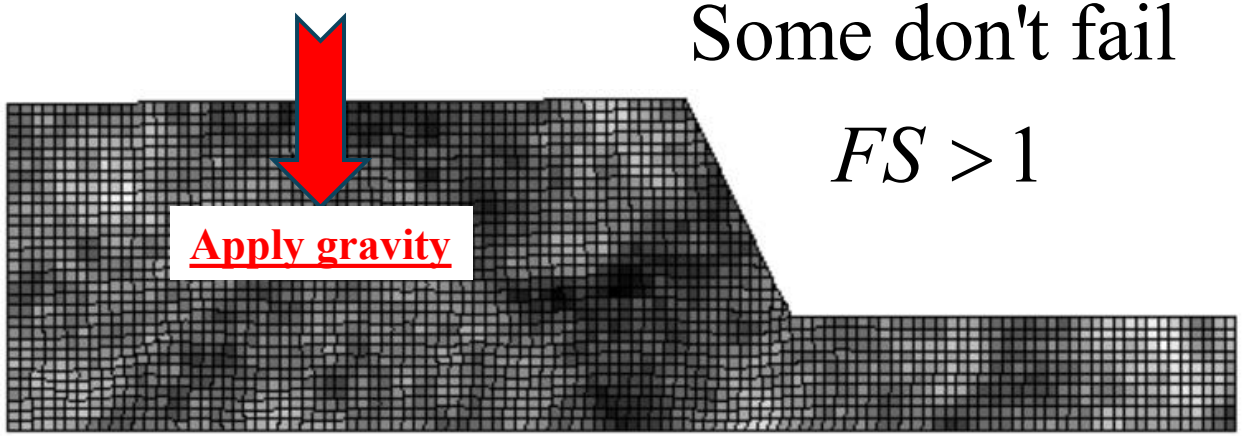
and for each simulation perform a slope stability analyses

Although the random field properties are the same for all simulations,

the weaker and stronger zones appear in different places, so

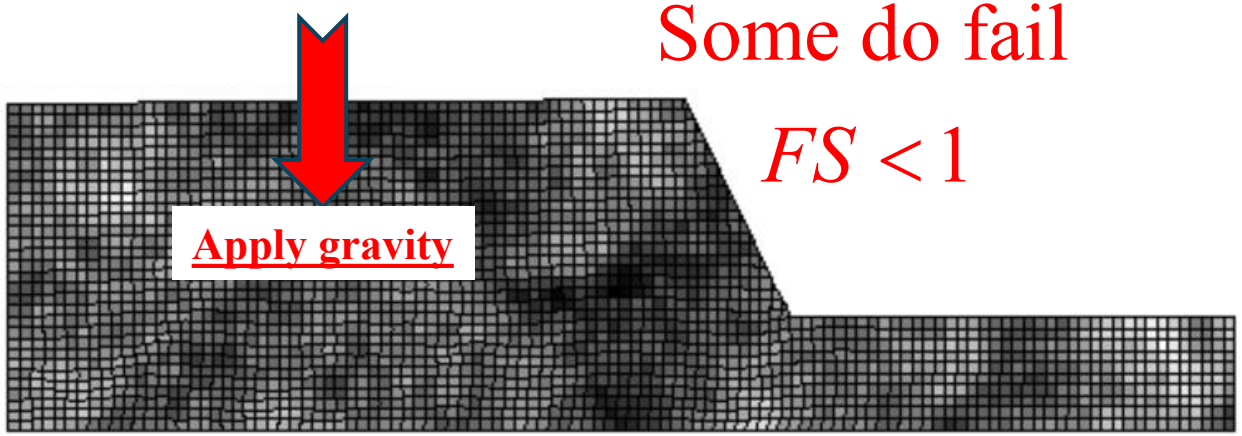
Some don't fail

$FS > 1$



Some do fail

$FS < 1$



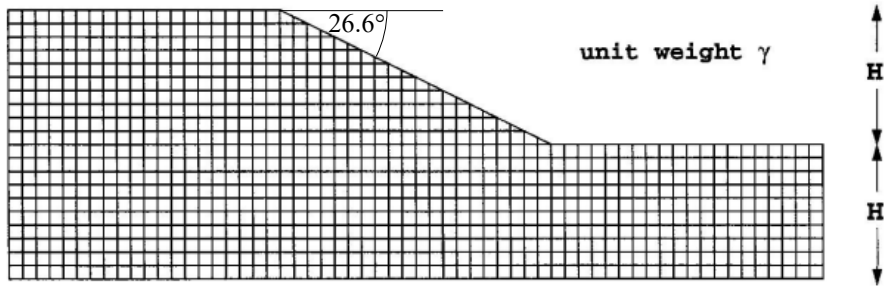
Keep count of the number of simulations that fail n_{fail}

The probability of failure is then simply obtained as $\boxed{p_f = \frac{n_{\text{fail}}}{N}}$ **output**

- **Worst-case spatial correlation length (p_f maximum)**

Consider a slope where β=26.6° << 53°

Well into the "flat" range of Taylor's chart



$$\mu_{FS} \approx 1.47$$

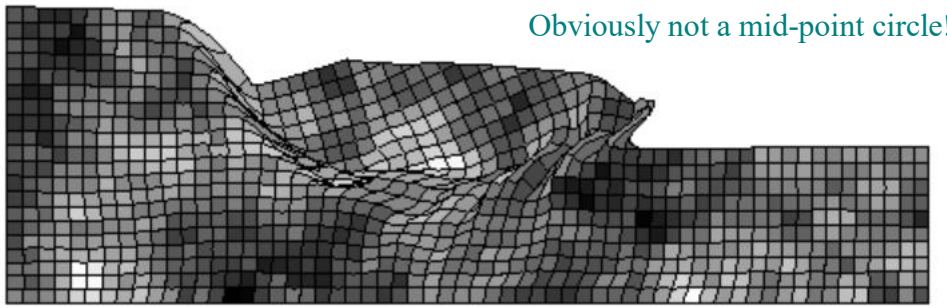
$$\mu_{s_u} = 50$$

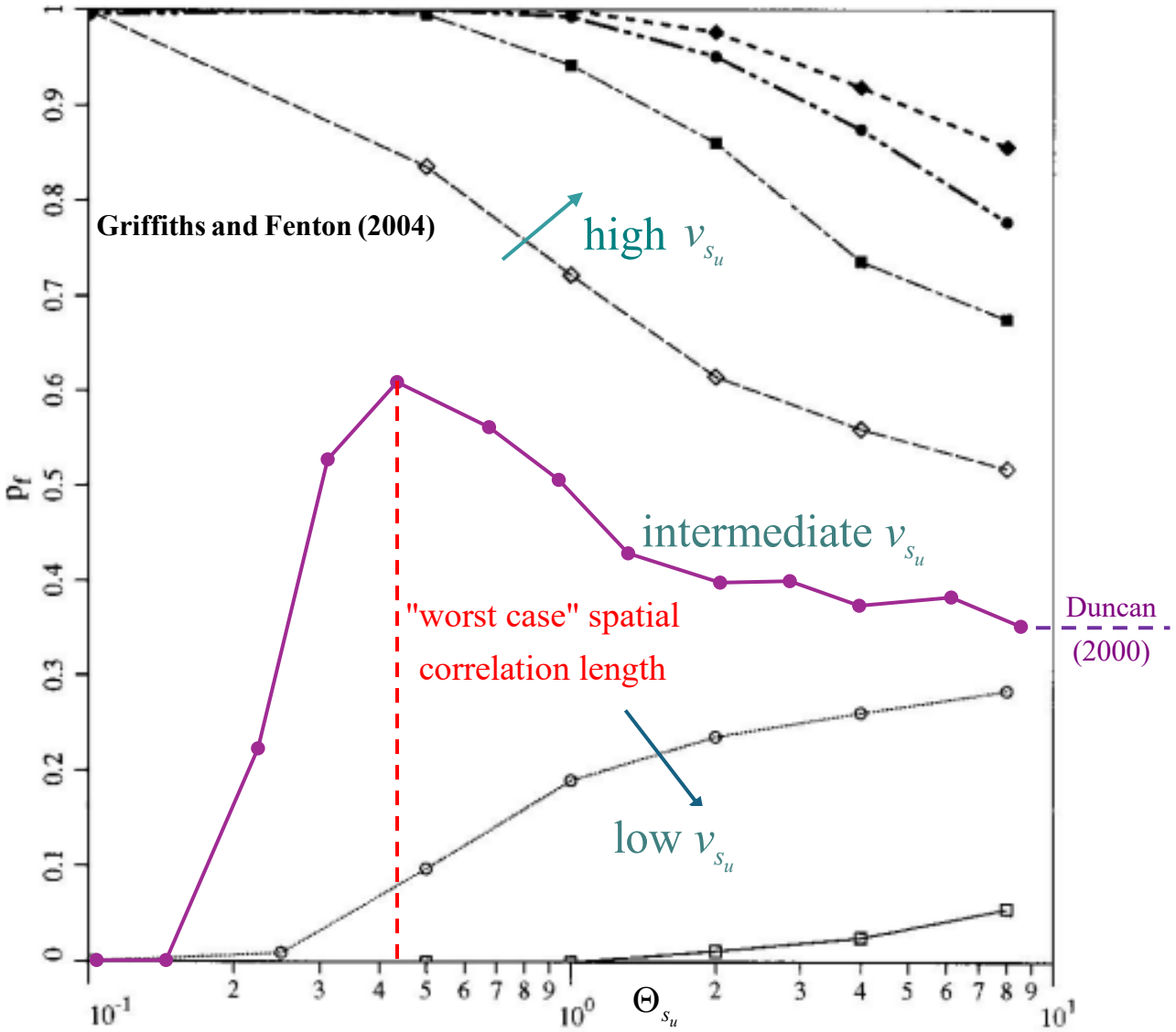
$$\sigma_{s_u} = v_{s_u}\,\mu_{s_u}$$

$$\beta = 26.6°$$

$$D = 2$$

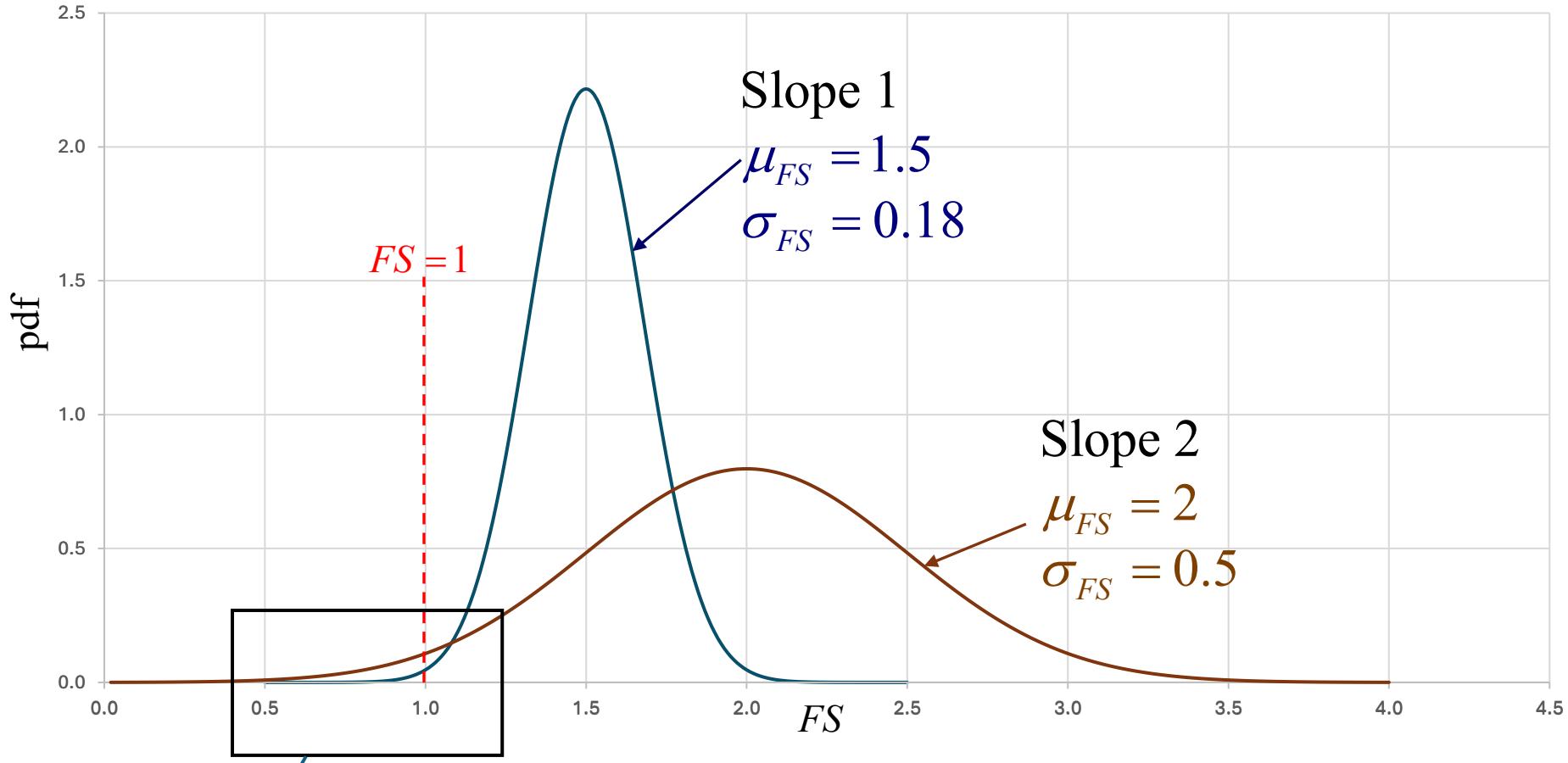
Typical failure mechanism
from RFEM slope stability analysis
Obviously not a mid-point circle!

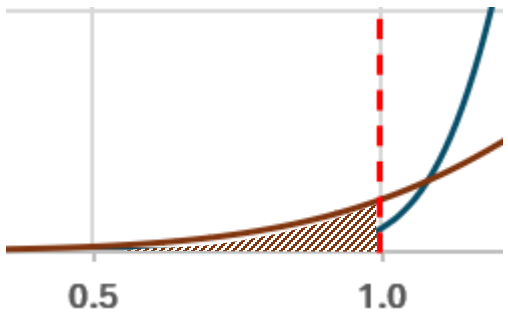




Failure to account for spatial variability could significantly overestimate or (worse) underestimate p_f

What, if any, is the relationship between *FS* and p_f?





Slope 2 with the higher mean *FS*, also has a higher p_f !

Use caution when comparing *FS* with p_f

The *FS* is based on "characteristic" values which do not account for variability

IV. CONCLUDING REMARKS

- ***Factor of Safety methods*** **have evolved over the past ~100 years to include:**
  a) **charts**
  b) **limit equilibrium methods**
  c) **numerical methods such as FE and FD**

- **Widely available numerical methods using strength reduction or limit analysis remain the most user-friendly, robust and accurate.**

- ***Probability of Failure*** **methods enable us to manage safety more scientifically:**
  a) **by focusing on those parts of the slope design that have the greatest uncertainty.**
  b) **by recognizing that the probability of failure is never zero.**
  c) **by deciding what *Probability of Failure* can be tolerated based on budget, and consequences of failure.**

- **Failure to account for spatial variability could lead to overestimation or, worse, underestimation of the *Probability of Failure***

- **In the medium term both methods will co-exist and complement each other as engineers become more comfortable with probabilistic methods.**

- **Take care comparing *FS* with p_f. *FS* takes no account of the uncertainty of input properties.**



Thankyou for your attention

and to numerous colleagues and students, past and present!

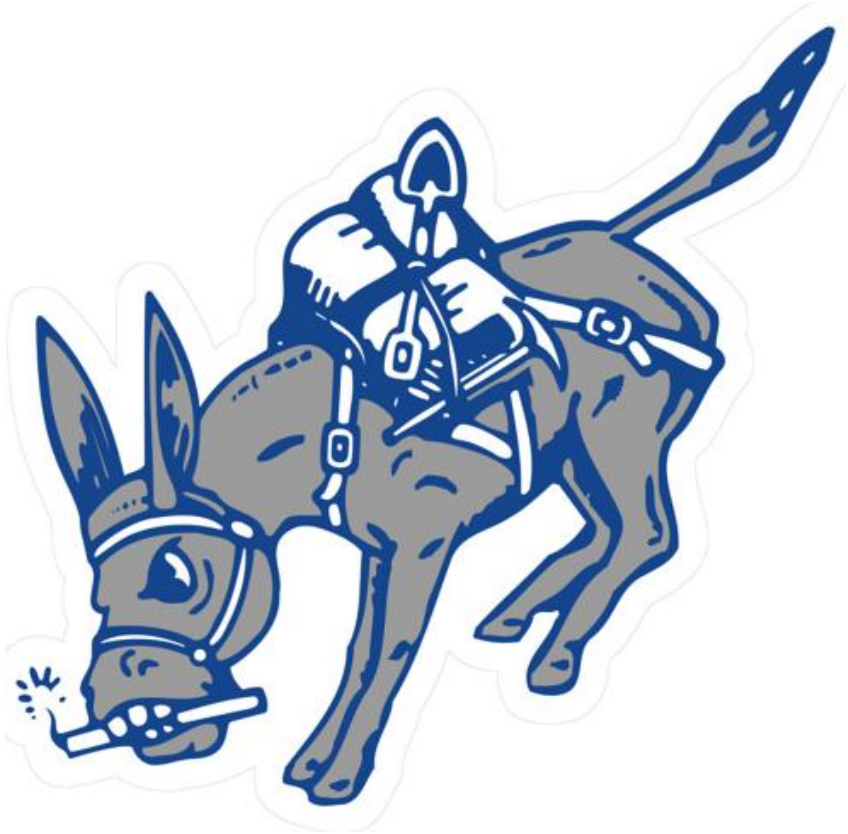


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The Jayhawk was originally designed as a mash-up of two real birds local to Kansas: the bold, raucous Blue Jay and the stealthy, fierce Sparrow Hawk (now known as the American kestrel)



Colorado School of Mines